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REMEMBERING PAUL ERDŐS

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The last time I saw Paul Erdős was in May 1995, during the Conference on Analytic Theory held in Urbana, Ill. in honour of H. Halberstam. It was a beautiful spring day, and we were walking (slowly, as Erdős had some trouble with his foot) in the park of Allerton Conference Center, where the meeting took place. At one point Erdős turned to me and said: "You know, it is said that that those who prove the prime number theorem will live forever. Well, Hadamard and de la Vallée–Poussin both lived to be almost a hundred. Now Selberg and I had also to do something with the prime number theorem, so...". The unfinished sentence hung in the air, full of understatement concerning his (and Selberg's) rôle in the proof of the prime number theorem. I don't remember now my exact reply, but the essence of it was that I told him he will live anyway through the myriad of his results.

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Unfortunately, Erdős died on September 20, 1996. I was, as must have been many of his friends and colleagues, utterly shocked and unprepared for the news of his death. I got to know of this sad event through an e-mail message from Carl Pomerance late on Sep. 21. He forwarded to me a message of Miki Simonovits from Budapest. In it was said:

"*Paul Erdős died Friday afternoon (20 September, 1996), in Warsaw. Early morning he felt some health problems, in a hotel in Warsaw. So he was carried into a hospital, where he died in the afternoon. (He was 83).*

As far as I know, he had a heart attack, rather serious, very early in the morning, in this Warsaw hotel, where he stayed while visiting the Minisemester for Combinatorics, (for two weeks), gave two lectures. Vera Sós and Andras Sarközy are leaving Budapest for Warsaw right now, Saturday morning, and Paul's original plans were to fly from Warsaw to Vilnius (for the Kubilius Conference) with Vera and Andras together, on Sunday.

The doctor informed us that he had two heart attacks and the second one killed him. (Perhaps even the first one was serious enough to prevent him from communicating to his surrounding. This may explain e.g. that he could not reach the mathematicians. We learned about his heart attacks only after his death.)"

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This text contains my personal reminescences of Erdős. There are certainly people who knew him better than I did, both professionally and even more so on the personal level, but I dare say I knew him well. Over the period of some twenty years or so, we were in contact, and the tangible result are eight papers that we jointly published (one was also coauthored by J.-M. De Koninck, one by C. Pomerance, and one by S.W. Graham and Pomerance). I most certainly have received more mail from him than from anybody else; it was easy to recognize his characteristic handwriting (in big, somewhat childish letters) on letters coming from all over the world. Sometimes it was even fun to guess where the letters would be coming from: Europe, North America, Australia, ... He would customarily sign them "Au revoir, E.P.", as in Hungarian the last name is usually put before the given name; he was born on March 26, 1913 as Erdős Pál (the "Paul" came later).

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Sometimes I lost track of where he was for quite some time. This was rather annoying, especially at times when we were doing joint work, and I simply did not know how to contact him. In particular, I could not locate his whereabouts at the beginning of this year, 1996. The Faculty of Mathematics of Belgrade University was about to organize, at the end of May, a three-day symposium in honour of Đ. Kurepa. Kurepa was an eminent Serbian mathematician, who died in 1993 at the age of 87. They knew each other quite well and Erdős, who visited Belgrade many times, regularly came to Kurepa's apartement to talk to him. When Kurepa died, Erdős in principle agreed to come to the symposium and give a conference in which he was to talk about Kurepa's work and his year-long relationship with him. Actually Ž. Mijajlović, the chief organizer, managed to find money for Erdős to stay a week in a decent hotel, which was remarkable in view of the bad economic situation in Serbia. After many efforts and a lot of e-mail messages, we located Erdős in California in April, if I remember correctly. Unfortunately it was already too late for Erdős to change his plans and come to Belgrade, moreover the trip from California to Belgrade would be rather tiring for a man of his age, though he traveled constantly. This I regret very much, for clearly it would have been my last chance to see him and talk to him, and besides his presence would have rendered the Kurepa symposium more attractive.

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Curiously enough, the last paper of mine that has just been published is "On the number of divisors of $n!$", which is joint work with Erdős, S.W. Graham and C. Pomerance. It appeared in "Analytic Number Theory: Proceedings of a Conference in Honor of Heini Halberstam, Volume I", pp. 337-355. The paper was actually completed during the Halberstam Conference in Urbana in May 1995. The work was done in a room at the Allerton Conference Center, during two evenings. What I remember most now is that the work was rather hard, one of the main task being to convince Erdős that we have done enough for the present paper, and should stop. Erdős, however, always had (as usual) new problems, new ideas, several generalizations. I think that he was doubtlessly the greatest problem proposer of all times. He also had the talent to judge what to propose to whom, and to pick problems suited to the capabilities of the potential solvers.

But the story with this paper does not end here. This August in Ottawa, during the CNTA-5 Conference (this stands for the "Canadian Number Theory Association", and this was the fifth of a series of meetings), C. Pomerance and I got together and worked on some unfinished problems related to the abovementioned paper. The problems were, in its original form, due to Erdős. Just as we were finishing a joint note, the sad news of Erdős's death came. I think that the dedication Pomerance put in our paper is a very appropriate one:

"Dedicated to the memory of our friend and teacher, Paul Erdős"

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For Erdős was exactly that: both a friend and a teacher, not only to Pomerance and myself, but to generations of number-theorists (and very many mathematicians working in other fields) from over the world. His "teaching" was rather subtle. He never directly gave lessons, taught theorems and methods. It was through *working* with him that one got a grasp of the richness of his methods and ideas. Slowly, perhaps even without noticing, he would take over in your mathematical way of thinking. He would painlessly convince you, by the success of his methods, what and how to use. In fact, many times when I was working alone on a problem, I would ask myself: "How would Erdős do it? How would he attack this problem?" It is sometimes subconsciously that I would do this, but as time went by I realized that I often *wanted* to try to do things his way, because it was the right way to do it. I've heard him say that God up in heaven has the one and only copy of **the BOOK**. This book contains the proofs of all possible mathematical theorems one could imagine, and moreover the proofs are the best and most elegant ones. When he said that a proof came closely to being **from the BOOK**, that meant high praise indeed.

Perhaps it is hard to explain, without going into technicalities (which I don't want), what were Erdős's methods. One of the principles was that the proof should be divided in cases, according to the size of the relevant quantities. To illustrate this, let me give a fictitious example (this includes all references to authors) of a result proved in, what I think, is the spirit of Erdős. In other words, this is how I believe Erdős would have proceeded. Thus consider the following

Theorem. *Call a natural number tolmonic if the sum of its tolmic parts does not exceed the sum of all its proper divisors. If F(x) is the number of tolmonic integers not exceeding x, then*

$$F(x) = O\left(\frac{x}{\log x}\right).$$

Proof. If n is tolmonic, let $P(n)$ denote its largest prime factor. Consider the cases

a) $P(n) < \exp(\sqrt{\log x})$,

b) $\exp(\sqrt{\log x}) \le P(n) \le \exp\left(\frac{\log x}{\log\log x}\right)$,

c) $P(n) > \exp\left(\frac{\log x}{\log\log x}\right)$.

The number of tolmonic n satisfying a) is $O(x/\log x)$ by Brun's sieve and estimates for $\psi(x, y) = \sum_{n\le x, P(n)\le y} 1$. For tolmonic n satisfying c) the same is also easily seen by applying a theorem of Henenbaum and Tall. Finally for those tolmonic n satisfying b), note that the defining property gives that n is Δ-distributed. But this implies that the sum of its proper divisors lies in an interval of the form $[\log^A, \log^B x], 0 < A < B$. Since the sum of tolmic parts of n does not exceed the sum of its proper divisors, by a result of Erdős from 1937 the number of such n is $\ll x/\log x$, unless $P(n) \le \log^C x$, but this is not possible by b), so the proof is complete.

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Erdős visited many times Sarajevo, of course before the war started. In the summer of 1992 he sent me the following message in one of his letters:

"Why war in Bosnia? Everybody loses."

There is no doubt that Erdős was right. He was a very compassionate man, and this showed (among other things) in the understanding of the hardships that the war created in Serbia (refugees, sanctions, negative image of Serbs in Western media, ...). In the summer of 1993, which was definitely the worst year, I had the pleasure of seeing him twice in Hungary. The first time was in June, and I came to Budapest one week before the Lillafüred conference on analytic number theory. Imre Kátai kindly provided the financing for my stay, but nevertheless I was tight for money. Erdős took me out for lunch in a nice restaurant near the Mathematical Institute in the Reáltanoda street. While discussing some problems involving the largest prime factor of n, Erdős suddenly pulled out of his jacket pocket a fistful of banknotes of money of several different countries. "Take it", he kindly said in a soft way, "I know you are hard up for foreign currency". I was quite embarassed, and told him that, having in mind the political and economic situation in Belgrade, I certainly could not repay the debt in the foreseeable future. Erdős insisted, but I managed to refuse, however the gesture I will never forget.

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Erdős had the unique talent of being able to say *something* concrete about almost any imaginable problem. Most mathematicians usually cannot say anything about a problem, and in a few instances they can say quite a lot. Erdős was exceptional, in the sense that he had the very rare gift of being able to "attack" almost any problem. Thus he was a living encyclopaedia in some sense, and I've heard often people say to each other: "If you don't know, ask Erdős". His memory also was, even in his last years, fascinating. He knew literally thousands of results, with rather precise bibliographical data. He was literally a mathematician "qui ouvre la voie" ("who opens the path"), as G. Tenenbaum put in a dedication to Erdős in one of his papers.

Once I was working with him on a problem involving squarefree numbers. We needed a certain result in the course of our investigations, and I did not know where to look for it. Erdős said: "There is a relevant paper by Mirsky in the "Quarterly J. Mathematics (Oxford)" in 1947 or 1948 at the latest". I went to the library and there it was, in all its semi-forgotten splendour: L. Mirsky, *Note on an asymptotic formula connected with r-free integers*, Quart. J. Math. Oxford **18** (1947), 178-182. Of course, this kind of situation occurred often, and I knew well when Erdős started the sentence with “There is a relevant paper...” that the paper in question was relevant indeed. And the additional beauty is that he very likely knew the author and probably had a joint paper or two with him.

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Erdős was truly prolific, and in the matter of the number of published papers he is probably without peer in the history in mathematics (some great mathematicians, like Euler, probably published more *pages* than Erdős did, but math was certainly different in Euler's days). Several years ago I asked him how many does he have, and he said: “$1200 \pm \varepsilon$”. By now the number has increased to about 1500 I believe, and tens of joint papers will appear posthumously. I pity the person(s) who will work on publishing his bibliography! The number of his collaborators must be well over 400, and Erdős told me that alone with A. Sárközy he has more than 50 joint papers. I suspect that it is Erdős himself who came up with the notion of the ”Erdős number”. This is defined as follows. Erdős had the number 0, those who had a joint paper with Erdős had Erdős number 1, those who had a joint paper with someone having Erdős number 1 had Erdős number 2 etc. It is, of course, a joke that every living mathematician who had a joint paper had a finite Erdős number, but certainly a very large number of mathematicians have a finite Erdős number. And it is truly fascinating when you start looking at some of the fields he worked on. He had papers from approximation theory, group theory, set theory, polynomials, numerical analysis - you name it, he sure has got it. For example, I was surprised that in the ‘50's Erdős wrote a joint paper with Jovan Karamata, the great Serbian mathematician. The paper was on summability methods, a field one usually does not associate Erdős with. When I asked him about this particular paper, since Karamata's work always interested me, he smiled and said that he knew Karamata quite well. He explained, in rather remarkable detail, the content of the paper, and then he went on and told me how he regularly visited Karamata in Geneva (where he was forced to emigrate after World War II by the Yugoslav communist goverment), told me stories about him, anecdotes ... As I already said, there is hardly a contemporary of his that Erdős did not know, and very likely he had a paper or two written with him.

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The language of Erdős (his English always had an unmistakable Hungarian accent, and he was fluent in French and German besides of course Hungarian, his mother tongue) was full of amusing idiosyncrasies. For example, ”noise” meant ”music”, ”poison” meant ”alcoholic drink”, ”boss” meant ”wife”, ”ϵ's” meant children. Besides, Uncle Joe and Uncle Sam stood of course for the Soviet Union and the USA, respectively. Naturally, when Erdős asked for an ϵ of wine, this meant a small, or symbolic quantity. He did not drink or smoke, and in fact disliked smoking. When I saw the effigy of A. Rényi in the hall of the Mathematical Institute of Budapest, I remarked to Erdős that Rényi died relatively young (49), and asked him for the cause of his death. He replied curtly “He smoked himself to death”. Obviously the very thought of this upset him, since he was close to Rényi. When he calmed down he explained then that Rényi died from lung cancer, which was most likely caused by excessive smoking. Perhaps his closest friend and collaborator was Paul Turán, who also died of cancer in 1976. Erdős often used to mention with kindness many mathematicians, but I think that it was Turán who occupied a special place in his heart.

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When a mathematician publishes the unbelievable number of 1500 papers, people are bound to ask questions. How was that humanly possible? Are the papers all really that good? Does he even remember what was in them? I will try to answer these questions, to the best of my knowledge. First, Erdős started very early publishing papers, when he was not yet even twenty, and continued to do so (and practically nothing else, since he never had a regular job to speak of) for well over sixty years. Besides, he was doubtlessly in the genius class (he discovered negative integers when he was merely three years old!). His working habits were formidable, he had no working hours, he worked (sometimes literally) day and night. I have the feeling that, when he was in his prime, he could have proved anything if he wanted to. If you compare an ordinary, successfull mathematician to a computer, then we should compare Erdős to a *transputer*, because he had the rare talent of working *parallel* on several problems at the same time. (He always reminded me in that aspect of the great Brazilian composer Heitor Villa-Lobos, who used to compose music while *listening to the radio play*). I've witnessed this: people would come to him, he would talk for several minutes (quite seriously) with a person about one problem, then with the next person about the second problem etc.

As for the quality of his papers, no, they are not all of the same level. Some are good, some are very good, and some are great. Erdős had this basic philosophy: if a result is new and correct, **publish it**. Now there are many very fine mathematicians who do not agree with this philosophy. They say no, a result has to be polished, great if possible to be published. They say one's papers should be in *increasing quality*, nothing to be changed, nothing to be added. This is fine with me, if it can be achieved. But I, having been "Erdősised" to the core, accept and adhere to Erdős's philosophy. Although Erdős never said this explicitly (at least not to me, and frankly I did not ask) I think I can pretty well get the idea behind his attitude about the publishing of papers. It is the very **work itself** that is, in a higher sense, the purpose of this whole game of mathematics. It is fulfilling in itself and brings meaning, and Erdős sacrificed his personal life (had no family, never married) to the research in mathematics. Now publishing was the visible, tangible aspect of the game, but the countless efforts, tries, conjectures, false leads, failures, heated discussions with his colleagues remain unseen (once, in Oberwolfach, I had a "heated" discussion with him that lasted more than an hour. People later told me they heard quite loud shouting coming from the room - God didn't give me a quiet voice - but after the verbal storm Erdős and I came out of the room the best of friends like we were before the discussion). And, of course, practice makes perfect even in writing papers and Erdős surely had lots of it. After a while writing papers becomes, in a sense, almost a habit ("addiction" would be perhaps too strong to say). As Erdős publishes, he throws seeds around. This is because often his papers lead to sharpenings, generalizations, new problems. And his seeds grow to new papers, new talents are being attracted to the game, and the whole process flows continuously for *decades*. "Prove and conjecture" was his credo. Certainly Erdős could have, with additional effort, improved many of his own results. I am quite sure that he did not want to do that, in many cases, on purpose, since it would take him valuable time for which he had other use.

As for the question how much he knew of his own theorems, I've already explained that he had an exceptional memory. Of course, he must have forgotten quite a few of his results, but in general I think he kept in his head much more of his own mathematics than people usually thought that he did.

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Let me get back to the fictitious Theorem about the mysterious "tolmonic numbers" (this I made up since it sounded vaguely familiar to me, but I sure can't remember where I got it from!). Anyway, what could have happened is the following. After having read the proof, L.J. Picolas observed that the problem could be made to depend on exponential sums. He proved that $F(x)$, the counting function of tolmonic numbers, actually satisfies $F(x) = O(x\log^{-A} x)$ for any fixed $A > 0$ by using simple estimates from the theory of exponential sums. Then by using the more sophisticated theory of exponent pairs Baker and Cook showed that $F(x) = O(x^{0.95})$ holds, and H. Nuxley announced further improvements. This pleased Erdős, who came up with the following flood of new questions:

a) Erdős can prove $F(x) = \Omega(\log^C x)$ for some $C > 0$. Is it true that $F(x) = \Omega(\sqrt{x})$? Is it true that $F(x) = O(x^{\frac{1}{2}+\varepsilon})$?

b) Let $1 = a_1 < a_2 < \cdots$ denote the sequence of tolmonic numbers. Is it true that $a_n^2 < a_{n+1}a_{n-1}$ with probability 1/2?

c) Is it true that $a_{n+1} - a_n \ll a_n^{1/2}$? (Erdős can prove it with exponent 11/12).

d) Is it true that $\sum_{n\le x}(a_{n+1} - a_n)^2 \ll x^{1+\varepsilon}$?

e) If $\omega(n)$ is the number of distinct prime factors of n, is it true that $\omega(n) = O(\sqrt{\log\log n})$ if n is tolmonic?

f) Is it true ... - but I better stop here, since by now you must have gotten the picture.

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As I said, some of his papers were truly great. I've asked him precisely three times what he thought were his greatest achievements in Number Theory. Each time he replied without hesitation: the following three things.

1. The proof that $\omega(n), \Omega(n)$ (the number of distinct and the number of all prime factors of n) obey the Gaussian normal distribution law. This is joint work with M. Kac, and is one of the crowning achievements of Probabilistic Number Theory (On the Gaussian law of errors in the theory of additive functions, Proc. Nat. Acad. Sci. U.S.A. **25** (1939), 206-207).

2. The characterization of the logarithm as an additive function and the work on the distribution of additive functions (On the distribution of additive functions, Annals of Mathematics **47** (1946), 1-20).

3. The elementary proof of the Prime Number Theorem (joint work with A. Selberg). This is: On a new method in elementary number theory which leads to an elementary proof of the prime number theorem, Proc. Nat. Acad. Sci. **35** (1949), 374-384.

Now the last paper is by Erdős alone, since it is well known that the elementary proof of the Prime Number Theorem developed into a **situation**. Since I was not a witness in 1948 of what really happened between Erdős and Selberg, since I haven't heard Selberg's version and since Erdős told me (twice: in 1981 and 1983) to keep confidential about his version I will not dwell on this topic here. After all, this is a matter for the history of mathematics.

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What I just said was corroborated in public by Erdős himself. In the summer of 1993 the János Bolyay Society organized a big conference in Keszthely (on the Balaton lake) in Combinatorics. The conference was, quite appropriately, in honour of Erdős's 80th birthday. Erdős himself was supposed to give a talk on his work. Well, since that was a conference on Combinatorics (I had a talk on combinatorial number theory, so this is how I attended) I thought he was going to talk primarily on his work in Combinatorics. After all, he is one of the founders of Graph Theory and much else besides, and what he had done in Combinatorics could hardly be squeezed into an one hour talk. However, all of us who listened to him (well over 200 people) were in for a big surprise. Actually there were *three* surprises. Erdős usually talked in the old-fashioned way, with blackboard, writing his formulas and thoughts with no notes. This time, he used an overhead projector and transparencies (after the talk, I snatched a couple of these - with his permission - as a souvenir, since he honoured me by quoting me at one place). The second surprise is that never, but I mean **never** have I heard him talk so clearly, lucidly, full of youthful vigour. At certain points during his lecture I was wondering whether this was really happening, whether an 80 year old man could give such a well organized and prepared talk. The third, and perhaps biggest, surprise is that practically 80 percent or so of his time was spent on his results from Number Theory, and not Combinatorics. Why did he do it? I have no idea, and I did not dare ask (actually in Keszthely it was hard to get physically close to him, there was literally a crowd of people around him all the time). Was it that he valued his results from Number Theory more than those from other fields, Combinatorics included? Perhaps that was the answer, since he clearly said that he considered as his greatest achievements

1. The proof that $\omega(n), \Omega(n)$ (the number of distinct and the number of all prime factors of n) obey the Gaussian normal distribution law. This is joint work with M. Kac (On the Gaussian law of errors in the theory of additive functions, Proc. Nat. Acad. Sci. U.S.A. **25** (1939), 206-207).

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I apologize for repeating this, but I consider this quite an important point, and this is why I did it.

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I've often asked Erdős to send me reprints of the ∞ of his published work. He did that many times, but nevertheless the number of his works that I have is far smaller than I would have liked. Erdős told me that his mother (both of his parents were teachers of mathematics) kept track of his published work until she died at the age of over 90. Actually almost until her death his mother accompanied him on his constant travels, in general organizing life for him. Erdős in fact owned an apartment in Budapest, but said he did not want to live in it when he was in Budapest, because his mother died there, and that brought on painful memories to him. So he seems to have let his friends use the apartment; in 1993 he told me Paul Turán's son lived there with his family. He stayed in a guest room of the Hungarian Academy of Sciences, whose prominent member he was for many years. It seems that the only salary that he received on a regular basis was from the Academy of Sciences in Budapest (the sum which, I believe, every academician receives) and from Haifa (Erdős was Jewish, but not in the strict sense) . The only catch is that, both in Budapest and in Haifa, money was available **only** when he was physically there. Otherwise, he would lecture at some University and get (very moderately) paid, or he would be invited as a guest to a conference (this happened a dozen times a year, I believe), or he would simply spend some time as a guest in some mathematician's home.

Erdős was truly cosmopolitan, certainly more than anyone else that I knew. He carried a Hungarian passport, in fact it was a sort of a diplomatic passport which he somehow managed to acquire long ago. It was often replaced by a new one, since he traveled so much that the visas of different countries would quickly fill it up. Although he was famous (I've heard that mail reached him simply if it had on the envelope "P. Erdős - Budapest") he often complained that he had trouble obtaining visas from certain countries, which used to complicate his travels.

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Of course Erdős was the prototype of the itinerant mathematician, who basically never stayed at the same place. He went on carrying very little personal belongings, which never really seemed to fill up a battered old suitcase. He dressed in silk shirts (his skin was sensitive to most other materials), usually wore sandals, and had a dark non-descript worn suit. His rooms were always filled with letters, manuscripts and a motley collection of coloured pills, which he swallowed with unmistakable regularity. "These are vitamins" he used to explain curtly, although I suspect there were surely other things that he must have been taking. When I complained several times that, in the interest of his health, he should be careful about what he was taking, he sneered and said that at his age nothing could harm him much. "Two worst things in life are old age and stupidity!" he used to say, and by this remark he managed always to end up a conversation.

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When we were together at a meeting and wanted to work, we had the following deal. I would come up to his room between 6 and 6.30 in the morning and we would work about 8, when usually breakfast was available. Since I am an early riser this, in principle, suited me, but it used to be tiring at certain times. The work was done as much as we could, before either one of us tired; then we would start talking about other things. First it was mathematics, then we would slip on to other subjects. Erdős was a remarkably well-informed man. He used to carry a big old transistor radio, which was strong enough to capture radio transmissions from Budapest from most parts of the globe, I presume. The reason for this is that he wanted to hear "as soon as possible" when/if the downfall of communism came, for which I had a full understanding. I have many memories from those early hours spent with Erdős. Once in Amalfi (Italy) in 1989, when I stayed up late the previous night, I completely forgot about the appointment with Erdős the next morning. But he didn't, and exactly at 6.30 the telephone rang. "Where are you?" simply said the well-known voice, a little irritated. I apologized and rushed to his room getting dressed in the hotel corridor. The chambermaid's "Mamma mia!" explains well what I must have looked like when I was passing her.

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Only once I lost temper with him. This happened in the summer of 1982, when he came to Belgrade by plane, on a very hot day. As we were driving from the airport to the city, he started asking questions. "Why did we have to wait so much for the luggage? Why do you have these containers overflowing with garbage? Why are the roads so badly paved? Why are we waiting so long for the street lights to turn green?" This avalanche of questions went for quite some time, and although the remarks were basically all in place I lost temper at one point and yelled at him: "There is no **why** in communism!" An uneasy silence crept in for some time, but the incident was quickly behind us.

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That same summer Erdős really scared me. Let me tell you the story. As it was terribly hot in my apartment, Erdős suggested we go to the nearby park where it was somewhat less unpleasant ("I must have fresh air!" he used to say). We took along my two year old daughter Natalija, since we couldn't leave her alone. Anyway, by the time we got to the park and had our notebooks ready for some problems on the distribution of $P(n)$, I remembered that there was a little chore I had to do. "Fine", Erdős said,"leave the child here with me for a few minutes, and we'll continue when you get back". With some hesitation I agreed, and rushed off. However, when I came back puffing and sweating some 10-15 minutes later, the park was empty. No Erdős, no Natalija. I felt sick in the stomach and looked around once, twice, three times. No one in sight. Panic was starting to take over, and dark thoughts of death, kidnapping, traffic accidents came to my head. I ran from one side of the park to the other – nothing. Finally, when I was already thinking of calling the police, from across the street I saw them. They were walking slowly, Natalija holding Erdős's hand, and they seemed to be talking (in what language I never knew!) and smiling at each other. In her other hand there was some candy Erdős apaprently bought for her, and they both seemed to be perfectly happy in each other's company. Fear gave place to relief, and I ran towards them and hugged them both. "You shouldn't have worried" Erdős said with a sly smile, "I have my ways with children." And he did; many times he gave me chocolates for my daughters (Emilija was born two years later, in 1984), always asked how were they doing in school etc. It is amazing that always he remembered their exact age, like he kept the data in a computer file in his brain.

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Besides arithmetic functions I had, from the late seventies, an interest in the theory of $\zeta(s)$. In fact, I became in some sense a "split personality", because my work is equally divided between what I call "Erdős theory" and zeta-function theory. One of the reasons for this is that, no matter how exciting, "Erdős theory" usually seriously interests only a limited number of people, whereas $\zeta(s)$ is one of the central themes of Analytic Number Theory. Erdős knew well of this. Not only did he approve ("Your work in the theory of $\zeta(s)$ is certainly more important than in arithmetic functions" he told me bluntly long ago) but he showed a remarkable interest and knowledge of zeta-function theory. I told him, of course, that I actually did not believe the notorious Riemann Hypothesis, RH, that all complex zeros of $\zeta(s)$ have real parts equal to $1/2$. He told me that P. Turán "probably more disbelieved the RH than he believed in it", although he did not know Turán's arguments. When we met, he often asked me about the newest results in the theory of $\zeta(s)$, and I was only happy to tell him as much as I knew. "What about the RH - still nothing?" (meaning still unsettled) he used to ask, with a slightly impish grin on his face. As for his own views about the RH, he did not divulge them, probably he believed the RH, being an optimist by nature. And besides approving of my working in the theory of $\zeta(s)$ he gave me, during a bad personal situation of mine, a piece of advice I will never forget. "Aleksandar", he wrote,"**there is no justice in Heaven or Earth, but there is certainly justice in MATHEMATICS! Thus try to be as good as you can in Mathematics**." And with this quotation of his I will finish these notes. When I close my eyes I will always remember him, walking in some deep thoughts, with glasses and graying hair, a slim, stooped figure in a worn suit and sandals, always ready to pour out formulas, ideas, problems... The world of Mathematics will be never the same without him.

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