You should work on this assignment in groups of two or three. Try to form different groups from the first time—the more different people you work with on mathematics, the greater your experience in seeing different approaches, and this will help you become a better teacher. On this assignment, you may also work alone without penalty, but I don’t recommend it. If in a group, hand in one write-up, with all of your names on it, in alphabetical order. Groups larger than three are not permitted. On this project, because of its computer nature, it is OK to seek help from others outside your group.

As you work on this project you’ll be doing everything—the geometric constructions and the calculations—with the Geometer’s Sketchpad software, not by hand.

We’ll talk about this project a little in class, to get you started. The in-class Geometer’s Sketchpad demonstration was also important in your understanding the software. You can also play around with the demos that come with the software, and use the on-line help. We’ll be only scratching the surface of what is possible with this wonderful tool.

Do not wait until the last couple of days. Feel free to come to my office for help if you are having difficulty or need clarification.

The software should be available on computers throughout the university, in particular in the basement of Kresge Library and in the ERL in the Education Building. (It might be called “GSP” on the computer.) An appendix in your textbook explains how to use it. I have a few copies of the manuals in my office, if you’d like to borrow one for a few days at a time. You can also buy your own copy of Sketchpad from the publisher if you wish (just type “geometer’s sketchpad” into a search engine); the student edition costs about $40.

For each of the three parts, play around with the problem as indicated. Then get the printouts needed to show what you have done, and hand in the printouts together with any additional answers to questions. You can write on the printouts using the software or by hand. Use the Print Preview command to allow you to fit each printout to the page before sending it to the printer. Each part counts 11 points.

If you wish, you can e-mail me your Sketchpad worksheets (or give me the files using a CD, disk, or flash drive), especially if you have done something animated for part 3. But also hand in the printouts in any case.

1. The 9-point circle theorem

A rather obscure theorem in geometry states the following. Draw any triangle. Find the orthocenter (the intersection of the altitudes) and the circumcenter (the intersection of the perpendicular bisectors of the sides). Let \( O \) be the midpoint of the segment joining these two points. Then there is a circle centered at \( O \) that passes through the following nine points: the midpoints of the sides, the points of intersection of the altitudes and the sides, and the midpoints of the segments joining the orthocenter and the vertices.
Your job is to illustrate this with Geometer’s Sketchpad. Draw a random triangle using the line segment tool. Construct the various segments and points mentioned in the theorem (this will take some thought—the constructions have more than one step). Label things well (use the text tool). Construct the circle and note that it seems to contain all nine of those points. Then use the pointer tool to drag the vertices around and see that no matter how things change, the circle continues to contain the nine points.

Hand in two printouts showing what you have done—the second printout will be the same as the first but with the vertices dragged so that a different shaped triangle results.

One note of caution: When using Geometer’s Sketchpad to find the various centers (e.g., the intersection of the altitudes), be careful not to ask for a point common to three segments. The program knows what you mean if you point to a place where just two lines intersect, but it gets confused if three lines all seem concurrent there.

2. An area problem

Use the Preferences selection on the Edit menu to set units to have 5-decimal-place precision. Draw a convex quadrilateral. Trisect the sides (you probably want to use the parallel line algorithm we discussed in class; you’ll probably want to use the Translate transformation to get equally spaced points that you need for that algorithm). Do a measurement and drag points around to make sure that you did it correctly. Join the corresponding points on opposite sides. The result is a (deformed) $3 \times 3$ “checkerboard”, with nine smaller quadrilaterals, of different sizes. Color them checkerboard style (in two colors), except make the middle one a third color; there will be four of each major color (say red and blue). Find the areas of these nine quadrilaterals, as well as the area of the original quadrilateral. Find a nice relationship between the total red area and the total blue area, and find nice relationships among these, the area of the middle quadrilateral, and the area of the original quadrilateral. Have GSP do all the arithmetic.

As always, move the original vertices around and see how the relationships you found are maintained. In fact, see what happens if the quadrilateral is not convex.

Hand in a printout showing the picture and the calculations, and state in words exactly what relationships you found. Hand in a second picture with the points dragged to different positions.

3. Math and art

Be creative! Use the tools the software provides to make something visually appealing, with lots of symmetry. Maybe something involving fractals. Maybe something animated. You could look at some of the samples that come with the program to get ideas. This part is really open-ended, so you can pretty much do what you want. The idea is to explore the software, especially the transformations (translations, rotations, reflections, dilations), and have fun. I am sure that almost everyone in this class has more artistic talent than I do, and I’m looking forward to seeing the results. I will award 5 points of extra credit to each of what I consider to be the best two papers in this part (“the decision of the judges is final”).