1. A soccer ball can be thought of as a polyhedron, some of whose faces are 5-sided, and the rest of whose faces are 6-sided. (Just imagine deflating the ball a little and flattening each face.) At each vertex there are two hexagons and one pentagon. In all, the ball has 12 pentagons. You now have enough information to determine everything about the polyhedron. If you wish, you can come look at the model of part of this polyhedron at the front desk. The hints for the various parts might also be useful hints for other parts.

   (a) (3 points) How many vertices are there in all? [HINT: think about the relationship between pentagons and vertices.]

   (b) (3 points) How many hexagons are there in all? [HINT: think about the relationship between hexagons and vertices, but be careful.]

   (c) (3 points) How many edges are there in all? [HINT: each polygon “contributes” five or six edges, but watch out for overcounting.]

   (d) (3 points) Use your answers to parts (a), (b), and (c) to confirm Euler’s formula for the soccer ball.
2. (20 points) State correct, clear, precise, complete definitions of FIVE of the following. Write OMIT on the two you wish me to ignore (otherwise, I'll grade the first five you answer).

(a) A circle.

(b) Lines $\ell$ and $m$ in space are parallel.

(c) $\triangle PDQ \cong \triangle FYI$

(d) Quadrilateral $ABCD$ is convex.

(e) A regular hexagon.

(f) An oblique square prism.

(g) A right circular cone.
3. (8 points) The intersection of a dihedral angle and a plane in space can be five different things, if I counted correctly. Identify with a word or phrase FOUR of these possibilities, and draw a picture to illustrate each case you mention.

4. (7 points) In the spirit of Assignment #1, describe verbally the following picture of three lines and one point in a plane, so that a person reading or listening to your description can reproduce it. Your description must be unambiguous. You may not use any words that refer to positions (e.g., below) or sizes of things (e.g., acute angle).
5. This problem concerns the sum of the measures of the angles in a convex \( n \)-gon.
   (a) (3 points) Give a formula for this quantity, as a function of \( n \).
   
   (b) (5 points) Prove your formula. You may use the fact that the sum of the angles measures in a triangle is \( 180^\circ \).

6. Remember that a network (graph) is said to have an Euler path (or be “traversable”) if there is a path through the network such that each arc (edge) is traversed exactly once. Recall, too, that a network is said to have a Hamilton path (or be efficiently travelable by a traveling salesperson) if there is a path through the network such that each vertex is visited exactly once.
   (a) (4 points) Draw an example of a network which has no Hamilton tour, and explain why it has none.

   (b) (4 points) Draw an example of a network which has a Hamilton tour but no Euler tour, and explain why it has no Euler tour.
7. (7 points) It is a theorem of geometry that the diagonals of a parallelogram are perpendicular if and only if the parallelogram is a rhombus. One of the homework exercises dealt with this theorem. Prove either the “if” part or the “only if” part of this theorem. Make it clear which one you are proving; do not prove both directions.

8. (7 points) Using only compass and straightedge, construct an isosceles triangle whose base is the segment $AB$ at the bottom of the page, and whose angle included between the congruent sides is congruent to angle $J$ shown below. [HINT: this will take more than one step. Think about it before plunging in. Either show your construction by making the markings quite clear, or else add a few words of explanation.]
Multiple choice. (32 points) Each of the following problems is worth 4 points for the correct answer, 0 points for an incorrect or no answer (so guess any you don’t know). Circle the letter of the best answer to each question. No work need be shown.

9. How many points are there on a line segment?
   (a) none       (b) one     (c) two     (d) a large finite number
   (e) an infinite number

10. Which of these is not a simple closed curve?
    (a) a circle    (b) a square   (c) the interior of a triangle
        (d) a concave pentagon   (e) a trapezoid

11. How many acute angles can a triangle have?
    (a) 0, 1, 2, or 3   (b) 0, 1, or 2   (c) 1, 2, or 3
        (d) 2 or 3       (e) 1 or 3

12. Recall that a Möbius strip can be formed by taking a long strip of paper, giving one end a half-twist, and then joining the ends by taping them. We observed that this strange surface has only one side and only one edge. What happened when we cut the strip all around, midway between the edges?
    (a) We obtained two smaller, separate Möbius strips.
    (b) We obtained two smaller, linked Möbius strips.
    (c) We obtained two smaller strips, only one of which has a twist.
    (d) We obtained one large Möbius strip, with a knot in it.
    (e) We obtained one large unknotted strip with four half-twists.

13. Which of these is not true about the figure below?
    (a) $\angle 3$ and $\angle 5$ are alternate interior angles
    (b) $\angle 1$ and $\angle 3$ are vertical angles
    (c) $\angle 3$ and $\angle 4$ are corresponding angles
    (d) $\angle 2$ and $\angle 5$ are complementary angles
    (e) $\angle 3 \cong \angle 4$ if and only if the top and bottom lines are parallel
14. Suppose seven points are collinear. How many different true statements of the form “x is between y and z” can we make, using the names of three of these points in place of x, y, and z?
   (a) 7  (b) 21  (c) 35  (d) 42  (e) 210

15. If ℓ and m are skew lines, how many different planes that contain ℓ are parallel to m?
   (a) none  (b) one  (c) two  (d) an infinite number
   (e) The answer depends on ℓ and m.

16. Which of these statements about regular polyhedra is true?
   (a) Although they are fascinating mathematically, these solids do not appear in nature.
   (b) Some of them have been discovered only recently, with the help of computers.
   (c) For each of them, the sum of the angle measures at each vertex equals 360°.
   (d) The dodecahedron is the only one with pentagonal regions for its faces.
   (e) Since a cube has eight vertices, it is sometimes called an octahedron.