Instructions: Work all 18 problems and clearly state the answers. Show your work. Also, don’t forget that you need to communicate effectively with your reader. Point values are indicated; the total is 110, but your score will count as if out of 100 (so you get 10 points of errors for free). A calculator is allowed (although totally unnecessary on this exam), but no books or notes. You will need a compass, protractor, and straightedge. Ask me if anything is unclear. Papers are due at 2:50 p.m.

1. (8 points) Using only compass and straightedge, construct in the middle of this page a non-square rectangle whose sides are congruent to the two segments shown at the bottom of the page. Make sure your construction is clear, from the markings in your picture and/or with some verbal explanation of the steps involved.
2. (16 points) State correct, clear, precise, complete definitions of FOUR of the following. Leave completely blank, or write OMIT on, the five you wish me to ignore (otherwise, I’ll grade the first four you answer).

(a) a sphere

(b) Lines ℓ and m in space are parallel.

(c) ΔPDAQ is obtuse. [Do not use the word “obtuse” in the definition.]

(d) Quadrilateral ABCD is convex.

(e) a regular pentagon

(f) a right prism

(g) a circular cone

(h) a trapezoid

(i) the circumscribed circle for ΔGWB
3. (4 points) In the spirit of Assignment #1, describe verbally the following picture of two planes and one line in space, so that a person reading or listening to your description can reproduce it. Your description must be complete and unambiguous.

4. (5 points) In the spirit of Assignment #1, draw an accurate picture of the following situation of one plane and two lines in space: One of the lines lies in the plane, and the other line intersects the plane at one point and is skew to the first line. Use dashed lines where appropriate to indicate portions of items that are hidden from view.
5. Using the plastic polyhedron frameworks pieces demonstrated in class, I formed a polyhedron consisting of six squares and eight equilateral triangles. At each vertex two squares and two triangles come together. Every square is surrounded by four triangles, and every triangle is surrounded by three squares. The model is at the front desk. If you wish, you may come look at it but not touch it.

(a) (4 points) How many edges are there in all? Explain your reasoning. [HINT: Each edge borders both a square and a triangle.]

(b) (5 points) How many vertices are there in all? Explain your reasoning. [HINT: One approach here is to use the given information, your answer to part (a), and Euler’s formula. An alternative approach would be to see how many vertices the polygons contribute, and then correct for the fact that each vertex lies on four polygons. Try both ways if you have time, and make sure your two answers agree!]
6. We had some theorems for congruence of triangles, such as SAS and ASA. This problem investigates congruence theorems for convex quadrilaterals.

(a) (4 points) Explain why there is no “SSSS” theorem in this setting. In other words, explain how there can be two convex quadrilaterals $ABCD$ and $EFGH$ such that $AB = EF$, $BC = FG$, $CD = GH$, and $DA = HE$, but the two quadrilaterals are not congruent. A picture will be helpful here.

(b) (6 points) Prove the following “SASSS” theorem: Suppose that in convex quadrilaterals $ABCD$ and $EFGH$ we know $AB = EF$, $\angle ABC \cong \angle EFG$, $BC = FG$, $CD = GH$, and $DA = HE$. Then $\angle CDA \cong \angle GHE$. [HINT: Draw a picture. You will need to introduce an appropriate extra line. There are basically two steps to the argument.]
7. Remember that a network (vertex-edge graph) is said to have an Euler circuit if there is a path through the network such that each arc (edge) is traversed exactly once and the path ends where it starts. Similarly, a network is said to have a Hamilton circuit if there is a path through the network, again ending where it began, such that each vertex is visited exactly once.

(a) (3 points) Draw an example of a network with six vertices and eight edges.

(b) (4 points) Determine whether or not your network has an Euler circuit. Explain why your answer is correct.

(c) (4 points) Determine whether or not your network has a Hamilton circuit. Explain why your answer is correct.
8. In this problem you will draw a triangle using a straightedge and protractor.

(a) (4 points) Draw $\triangle ABC$ such that $m(\angle CAB) = 37^\circ$ and $m(\angle CBA) = 65^\circ$.

(b) (3 points) Determine the exact measure of $\angle ACB$.

Multiple choice. (40 points) Each of the following problems is worth 4 points for the correct answer, 0 points for an incorrect or no answer (so guess any you don’t know). Circle the letter of the best answer to each question. No work need be shown in this section.

9. How many points are there on a line segment?
   (a) none  (b) one  (c) two  (d) a large finite number  
   (e) an infinite number

10. Which of these is not a simple closed curve?
    (a) a circle  (b) a square  (c) the interior of a triangle 
    (d) a concave pentagon  (e) a trapezoid
11. Which of the following figures is not possible? [This questions comes from the Grade 5 ISAT, which is the analog of the MEAP in Illinois.]
   (a) a quadrilateral with four right angles
   (b) a triangle with congruent sides
   (c) a trapezoid with two right angles
   (d) a triangle with two right angles
   (e) a right scalene triangle

12. Recall that a Möbius strip can be formed by taking a long strip of paper, giving one end a half-twist, and then joining the ends by taping them. We observed that this strange surface has only one side. What happened when we cut the strip all around, midway between the edges?
   (a) We obtained two smaller, separate Möbius strips.
   (b) We obtained two smaller, linked Möbius strips.
   (c) We obtained two smaller strips, only one of which has a twist.
   (d) We obtained one large Möbius strip, with a knot in it.
   (e) We obtained one large unknotted strip with four half-twists.

13. Which of the following statements is not always true?
   (a) The figure obtained by joining the midpoints of the sides of a convex quadrilateral in order is a rhombus.
   (b) A square is a rhombus.
   (c) The diagonals of a kite intersect at right angles.
   (d) A quadrilateral in which opposite sides are congruent is a parallelogram.
   (e) A square is a rectangle.

14. Which of these statements about regular polyhedra is true?
   (a) Although they are fascinating mathematically, these solids do not appear in nature.
   (b) Some of them have been discovered only recently, with the help of computers.
   (c) For each of them, the sum of the angle measures at each vertex equals 360°.
   (d) The dodecahedron is the only one with pentagonal regions for its faces.
   (e) Since a cube has eight vertices, it is sometimes called an octahedron.
15. Which of these is not true about the figure below?
   (a) \( \angle 3 \) and \( \angle 5 \) are alternate interior angles.
   (b) \( \angle 1 \) and \( \angle 3 \) are vertical angles.
   (c) \( \angle 3 \) and \( \angle 4 \) are corresponding angles.
   (d) \( \angle 2 \) and \( \angle 5 \) are complementary angles.
   (e) \( \angle 3 \cong \angle 4 \) if and only if the top and bottom lines are parallel.

16. If \( \ell \) and \( m \) are skew lines, how many different planes that contain \( \ell \) are parallel to \( m \)?
   (a) none   (b) one   (c) two   (d) an infinite number
   (e) The answer depends on \( \ell \) and \( m \).

17. Angela wants to make a cylinder from paper shapes. What shapes will she need? [This question comes from the Grade 5 ISAT, which is the analog of the MEAP in Illinois.]
   (a) 2 circles and 1 rectangle
   (b) 4 triangles and 1 square
   (c) 2 triangles and 1 rectangle
   (d) 2 hexagons and 2 rectangles
   (e) 3 circles and 3 triangles

18. \( \triangle PQR \) is similar to \( \triangle STU \). What is the length of \( TS \)? [This question comes from the Grade 5 ISAT, which is the analog of the MEAP in Illinois.]

   (a) 4 cm   (b) 14 cm   (c) 16 cm   (d) 26 cm   (e) 32 cm