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Abstract

This paper defines a theory of letter and word perception and learning that takes the form of a first-order predicate logic. The proposed model differs from other feature-based theories by employing logical unification as the ultimate feature matching mechanism, rather than set theoretic operations or spreading activation. According to the model, a perceptual system is a complex composed of a sensory system and a cognitive system. The sensory system transforms visual stimuli into sensory representations or "icons." The cognitive system matches sensory icons with cognitive counterparts stored in long-term memory that may include more or less information than was present in the sensory icon. When a sensory icon includes more information than is present in its cognitive match, the cognitive system is subject to modification through unification-based learning. When a sensory representation includes less information than is present in its cognitive match, unification-based matching introduces extra cognitive information that is cognitively indistinguishable from the original sensory information. Unification-based matching appears to offer explanatory mechanisms both for top-down perceptual influences such as context effects, and for a variety of observations educators have noted in early reading acquisition.

Theories of letter and word perception often make the assumption that letters and words are composed of still more elemental perceptual features. By this account, a letter is made up of a set of oriented features, and words are composed of sequences of letters or feature sets. This conceptualization of letters and words as composed of simpler feature elements has had longstanding and nearly universal acceptance in the reading research community (e.g., Gough 1985; Massaro 1984; McClelland & Rumelhart 1981; Rumelhart & Siple 1974; Smith 1982). The feature-based account seems plausible, is in keeping with a scientific tradition of analysis and reduction, and is provided with some indirect support from research in the physiology of perception (Hubel & Wiesel 1979). Most important of all, however, the feature approach to letters and words provides the starting point for nearly every currently active theory of letter and word recognition in the literature of reading research.
Despite the attention feature theories have attracted over the past 100 years and the voluminous body of data that has been generated, most attempts at formalization of feature theory have usually focused on specific empirical questions (e.g., Massaro 1984; Oden 1979). The one comprehensive formalization of feature theory familiar to reading researchers is that of McClelland and Rumelhart (1981) which, as a connectionist model, suffers from a number of limitations noted by Fodor and Pylyshyn (1988). The purpose of the present paper is to propose an alternative formalization of feature theory based on first-order predicate logic. The approach taken in the present research is axiomatic (Stoll 1961, 1979). Primitive concepts, principles, and axioms are specified, and higher-order concepts are then defined in terms of these more basic notions.

On the Relevance of Formal Methods in Reading Research

Formal theories have two distinct advantages over more traditional non-formal theories. One advantage is that the explicit character of a formal theory allows it to be implemented relatively easily as a computer simulation. One recently developed logic-programming language, Prolog, (Clocksin & Mellish 1987; Sterling & Shapiro 1986) provides an ideal programming environment for the development of computational implementations of formal logic-based theories. A computational version of the formal theory described in this paper has been developed in Arity/Prolog (Arity Corporation 1986) and will be shown (in Part 2 of this paper), to be consistent with a wide variety of empirical findings described in the literature of letter and word recognition including: word superiority effects (Cattell 1885, 1886; Reicher 1969), pseudoword effects (Massaro & Klitzke 1979; McClelland & Johnston 1977; Zeitler 1980), visual factor effects (Johnston & McClelland 1973; Massaro & Klitzke 1979), repetition effects (Feustel, Shiffren & Salasoo 1983; Johnston, van Santen, & Hale 1985; Salasoo, Shiffren, & Feustel 1985), and characteristics of the letter and word recognition abilities of beginning readers.

A second advantage of developing formal theories is that a theory of this kind reveals underlying logical structure in a more adequate way than traditional intuitive theories. For example, a formal theory makes assumptions an explicit part of the theoretical apparatus. In addition, a formal theory in the form of a first-order predicate calculus provides a basis for developing a more general meta-theoretical framework for exploring theories in a general way.

Metaphysical Foundations

A formal theory can be conceptualized as a set of abstract objects and operations that can be applied to those objects. The objects of interest in the present research are all related to a certain account of perception. These objects are of three kinds.

A material object is a physical object. In the present circumstances, the material objects of interest are physical tokens that represent characters, letters, and words.

A sensory object is the representation of a material object by a sensory system. Sensory objects have traditionally been referred to as "icons" or "images." An example of a sensory object is the after-image that occurs when a very bright stimulus is briefly exposed. The subjective experience of an image that persists for a short while after the stimulus has been removed is usually thought to represent the gradually decaying sensory representation.
A cognitive object is also a representation, but unlike sensory objects that represent material objects, a cognitive object represents one or more sensory objects that are a part of the perceptual history of the perceptual system. Cognitive objects also differ from sensory objects in being long-term representations. Whereas sensory objects have a very brief "half-life" in the absence of the material objects they represent, cognitive objects can persist unchanged over long periods of time. Cognitive objects are, in effect, the patterns represented in long-term memory that allow a perceptual system to distinguish between different kinds of sensory objects.

The theory proposed here is a sense-datum theory of perception (Audi 1988). Sense-datum theories are characterized by a sensory mechanism that mediates between material objects and the cognitive representations of those objects in a cognitive system. The sense-datum approach adopted here distinguishes three domains that correspond to the three kinds of objects identified above.

The material domain corresponds to the notion of the "external world" and is defined by material objects (i.e., physical characters and spatial arrangements of characters). The sensory domain corresponds to the world of sense-data and is defined by the sensory objects created by a sensory system, usually in the transformation of a material input into a sensory representation of that input. The cognitive domain corresponds to the knowledge a perceptual system has that allows it to identify a sense-datum with some past sensory experience(s). The cognitive domain is defined by cognitive objects that are created and stored by a cognitive system over the course of its perceptual history.

The theory of letter and word perception proposed in this paper is developed in a formal axiomatic manner. In an axiomatic approach certain objects and relations are designated as primitives and certain principles and axioms are assumed. Other higher-order objects and operations are defined in terms of the primitives, and other principles are deductively derived from the axioms. The theory as a whole is founded upon its primitives and axioms in the same way that Euclidean geometry is founded upon the primitive notions of a point and a line, and the Euclidean postulates. It is natural, therefore, for the specification of a formal theory to begin with axioms and primitives.

Specification of the Formal Theory

The present research presumes six axioms that relate the system developed in this paper to an empirical interpretation. A foundational axiom \([AX_0]\) specifies theoretical primitives and identifies them with one (foundationalist position) or more (coherentist position) sets of visual features. The present theory adopts a foundationalist stance, where the set of primitives \(P\) is defined as \(\{x : x\text{ is a feature primitive}\}\), and feature primitives are defined as in \([D_{14}]\). A coherentist position is also possible, where the set \(P\) is not defined as one specific set of features.

The axiom of featural coherence \((AX_1)\) is the perceptual equivalent of the law of excluded middle. According to \([AX_1]\) features can assume only one of two mutually exclusive values (i.e., "on" (fn) or "off" (an)). The axiom of featural coherence is introduced by the requirement that every element in a character sequence satisfy the compatible feature set requirement, as defined in \([D_{15}]\). One alternative to the axiom of featural coherence has been proposed by Oden (1979) through the application of fuzzy sets rather than standard set theory.

The axiom of sensory transform \((AX_2)\) adopted in the present research assumes sensory performance limitations. \([AX_2]\) is defined in \([D_{29}]\) where the \(subset\) relation is
identified as the basis for generating sensory transforms. Since a subset may include fewer features than the original set, it is this axiom that introduces what is referred to as "sensory degradation."

The axiom of featural independence ([AX3]) is introduced indirectly in [D34], where the probability of a given sensory icon being produced from a visual stimulus is stated to be a simple product of the sampling probability of each feature in the stimulus. Some theorists (notably Smith 1982) seem to make claims for an axiom of featural dependence, but the present research suggests that featural dependence would be difficult to implement and may offer little in the way of additional explanatory power. Another aspect of the axiom of featural independence is its designation of the sensory system as a self-contained apparatus that operates independently of cognitive influence.

The axiom of cognitive matching ([AX4], specified in [D40]) introduces unification as the ultimate pattern matching mechanism in the model. Alternatives to unification-based matching include exact-match, subset-match, and fuzzy-set match criteria, as well as non-symbolic spreading activation. Finally, the axiom of cognitive development ([AX5], defined in [D43]), introduces the concept of unification-based learning. [AX5] distinguishes the present research from other theories of letter and word perception by emphasizing the role of perceptual learning in letter and word recognition.

A material feature primitive is interpreted as representing the presence or absence of a specific visible mark. A presence-feature is interpreted as the unambiguous presence of one, and only one, of the 14 specific types of visible marks that make up the Rumelhart and Siple font depicted in Figure 1a. Each of these 14 feature-presence primitives is represented by a lowercase "f", followed by an integer from 0 to 13, and superscripted with a lowercase "m." The 14 feature-presence primitives correspond to the 14 features that make up the Rumelhart and Siple (RS) font depicted in Figure 1a. The set of 14 feature-presence primitives is denoted by the set \( F_p^m \), where

\[
[D1] \quad F_p^m = \{f_0^m, f_1^m, f_2^m, f_3^m, f_4^m, f_5^m, f_6^m, f_7^m, f_8^m, f_9^m, f_{10}^m, f_{11}^m, f_{12}^m, f_{13}^m\}.
\]

Feature-presence primitives are governed by a uniqueness principle [UP1].

\[
[UP1] \quad (i)(j)(i \neq j) \rightarrow (f_i^m \neq f_j^m)).
\]

One way to represent a letter or character is simply to specify the set of features that are known to be present. According to this presence-only specification the letter "C" in the RS font is specified by the feature set \( c = \{f_2^m, f_5^m\} \). Under conditions of imperfect sensation or incomplete knowledge, however, the specification of characters solely on the basis of presence-features leads to undesirable ambiguity. It is important to distinguish the difference between the absence of knowledge regarding the presence of a feature and the knowledge that a feature is absent. The use of presence-only feature specification obscures this important distinction and, for this reason, characters are represented in a form that specifies both presence-features and absence-features. Accordingly, the letter "C" in the RS font is specified by the feature set:

\[
c = \{f_0^m, f_1^m, f_3^m, a_3^m, a_4^m, f_5^m, a_6^m, a_7^m, a_8^m, a_9^m, a_{10}^m, a_{11}^m, a_{12}^m, a_{13}^m\},
\]

where the known absence of a presence-feature \( f_n^m \) is denoted by a corresponding absence-feature \( a_n^m \).
Each feature-absence primitive is interpreted as representing the absence of a specific visible mark and is represented by a lowercase "a" followed by an integer between 0 and 13 inclusively with an "m" superscript. The fourteen feature-absence primitives that make up the feature-absence set $F_a^m$ is denoted as,

$$[D2] \quad F_a^m = \{a_0^m, a_1^m, a_2^m, a_3^m, a_4^m, a_5^m, a_6^m, a_7^m, a_8^m, a_9^m, a_{10}^m, a_{11}^m, a_{12}^m, a_{13}^m\}.$$  

Feature-absence primitives are also assumed to be governed by a uniqueness principle ([UP2]) such that,

$$[UP2] \quad (i)(j)((ij < 14) \rightarrow ((i \neq j) \rightarrow (a_i^m \neq a_j^m))).$$

A distinctness principle denotes $F_p^m$ and $F_a^m$ as disjoint sets.

$$[DP1] \quad F_p^m \cap F_a^m = \emptyset.$$

A sensory feature primitive will refer to the objects that result when a sensory system transforms material primitives. Since in the present context, 28 material primitives have been defined (14 presence-features and 14 absence-features), 28 sensory features are also assumed to exist. Thus,

$$[D3] \quad F_p^s = \{f_0^s, f_1^s, f_2^s, f_3^s, f_4^s, f_5^s, f_6^s, f_7^s, f_8^s, f_9^s, f_{10}^s, f_{11}^s, f_{12}^s, f_{13}^s\},$$

and

$$[D4] \quad F_a^s = \{a_0^s, a_1^s, a_2^s, a_3^s, a_4^s, a_5^s, a_6^s, a_7^s, a_8^s, a_9^s, a_{10}^s, a_{11}^s, a_{12}^s, a_{13}^s\}.$$
Like material feature primitives, sensory feature primitives are assumed to be governed by uniqueness and distinctness principles similar to [UP1], [UP2], and [DP1], but with superscripts appropriately modified.

Cognitive feature primitives are objects that make up the recognizable patterns constructed (and revised) by a cognitive system over the course of its perceptual history. In effect, cognitive features represent the “content” of long-term perceptual memory. Cognitive features are stipulated in a manner similar to material and sensory feature primitives.

\[ D5 \quad F_p^c = \{ f_0^5, f_1^5, f_2^5, f_3^5, f_4^5, f_5^5, f_6^5, f_7^5, f_8^5, f_9^5, f_{10}^5, f_{11}^5, f_{12}^5, f_{13}^5 \} \]

\[ D6 \quad F_s^c = \{ a_0^5, a_1^5, a_2^5, a_3^5, a_4^5, a_5^5, a_6^5, a_7^5, a_8^5, a_9^5, a_{10}^5, a_{11}^5, a_{12}^5, a_{13}^5 \} \]

Like material and sensory primitives, cognitive primitives are governed by uniqueness and distinctness principles.

It will frequently be useful to refer to features in a general way, without specifying whether the features intended are material, sensory, or cognitive in nature. When features are referred to in this general way, or when the context of use makes it obvious which kind of features are being referred to, un-superscripted feature notation is employed. Thus, the members of \( F_p^m \) will not be written \( \{ f_0^m, f_1^m, f_2^m, \ldots, f_{13}^m \} \) when it is clear from the context that the features referred to are material features. When the context makes the domain clear, or when features are referred to in a general way, un-superscripted feature notation is employed. For example, \( F_p^m \) is represented simply as \( \{ f_0, f_1, f_2, \ldots, f_{13} \} \). In circumstances where distinctions between material, sensory, and cognitive features are important superscripted feature notation is used.

The definitions that follow are presented in un-superscripted feature notation and, therefore, actually represent definition schemata that can be interpreted in three different ways, as applying to either material, sensory, or cognitive objects. The purpose of treating definitions in this general way is to develop three parallel models of the formal apparatus developed in this paper, one for each of the domains of interest.

One model is interpreted to apply to the physical objects we call printed material. The second model is interpreted as applying to the theoretical sensory objects (sensory images, icons, etc.) that have traditionally been described in the literature of perception. The third model is interpreted as applying to the theoretical cognitive objects (mental images, memories, knowledge, etc.) that have traditionally been described as making up the content of long-term perceptual memory. These three models will ultimately be integrated into a single perceptual model that incorporates all of the entities of the three respective domains and specifies relations that hold between objects in the three domains.

The Rumelhart and Siple (1974) Font

The Rumelhart and Siple font is defined as a set \( RS \) of sets of material feature primitives where,

\[ D7 \quad RS = \{ a, b, c, \ldots, x, y, z \} \]

and each of the bold lowercase letters that make up \( RS \) represents a unique set of feature primitives. For example,
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[D7a] \( a = \{a0, f1, f2, f3, f4, a5, f6, a7, f8, a9, a10, a11, a12, a13\}, \) and

[D7b] \( b = \{a0, a1, f2, f3, f4, f5, a6, f7, f8, a9, a10, a11, a12, a13\}. \)

In a similar fashion, every other letter of the RS font is represented by a unique identifying set of material feature primitives, as specified by McClelland and Rumelhart (1988, pp. 210, 225). It is also convenient to adopt the terminology specified below in [D8] - [D14].

The universal set for each of the three domains (material, sensory, and cognitive) is recursively defined as the set composed of those objects that are feature primitives or can be constructed of feature primitives according to the definitions provided below. Thus,

[D8] \( U^m = \{x : x \) is a material feature primitive or can be constructed from material feature primitives by the definitions in this paper\}, \)

[D9] \( U^s = \{x : x \) is a sensory feature primitive or ...\}, \)

[D10] \( U^c = \{x : x \) is a cognitive feature primitive or ...\}, \) and

[D11] \( U = U \{x : x \in \{U^m, U^s, U^c\}\}. \)

[D12] \( x \) is a feature-presence primitive \( \leftrightarrow x \in F^p \).

[D13] \( x \) is a feature-absence primitive \( \leftrightarrow x \in F^a. \)

[D14] \( x \) is a feature primitive \( \leftrightarrow x \in U \{x : x \in \{F^m, F^s, F^c\}\}, \) where \( F_u = F^p \cup F^a. \)

Compatible Feature Sets

With the specification of primitives above, it is now possible to identify a set of objects that are of special interest in the present research: compatible feature sets and compatible feature set sequences. A compatible feature set \( F \) is any set of feature primitives that satisfies the following definition:

[D15] \( F \) is a compatible feature set (cfs) \( \leftrightarrow \)

\( F \subseteq 14 \land (F = F^m \lor F \subseteq F^s \lor F \subseteq F^c) \land (x)(x \in F \rightarrow (\exists n)((x = fn \land \lnot(an \in F)) \lor (x = an \land \lnot(fn \in F))). \)

Line 1 of the definiens requires that every cfs have no more than 14 elements, and that every element of a cfs belong to the same domain. Line 2 specifies that the 28 features identified can be divided into 14 distinct incompatible feature pairs \{fn, an\}. In other words, a feature can never be simultaneously present and absent. Note that every element of RS is a cfs.
Full and Partial Specification

Another important distinction is that of full specification and partial specification. Formal definitions for a fully specified cfs and a partially specified cfs are as follows:

[D16] \( F \) is a fully specified cfs \( \leftrightarrow \) \( F \) is a cfs & \( \text{Card}(F) = 14 \), where \( \text{Card}(F) \) is the number of elements in set \( F \).

[D17] \( F \) is a partially specified cfs

\[ \text{F is a cfs} & \text{ and } F \text{ is not a fully specified cfs.} \]

The term "CFS" will hereafter be used to refer to the set of all cfs (where cfs serves as the plural as well as the singular form for cfs). "CFS_f" and "CFS_p" is used, respectively, to refer to the sets of all cfs that are fully and partially specified.

The definitions provided to this point provide a basis for distinctness, completeness, and ordering principles that can be shown to apply to sets of cfs. [DP2] and subsequent related principles are referred to as distinctness principles. Each distinctness principle makes the claim that the sets of objects with which it is concerned are disjoint; that these sets do not overlap. [DP2], for example, states that there are no fully-specified cfs that are also partially-specified cfs, and vice versa.

[CP1] and subsequent related principles, are referred to as completeness principles. A completeness principle states that some set is composed of the union of two or more other, previously defined sets. [CP1] states that when the set of fully-specified cfs is combined with the set of partially-specified cfs the resulting combination (union) is a set that includes every cfs.

[OP1] and subsequent related principles are referred to as ordering principles. An ordering principle states that some set of objects can be ordered by a relation. [OP1] states that the set CFS can be ordered by the pollence relation \( \preceq \). In the present circumstances, the pollence relation is interpreted as a measure of the relative degree of completeness of the feature descriptions for two cfs.

[DP2] \( \text{CFS}_f \cap \text{CFS}_p = \emptyset \).

[CP1] \( \text{CFS}_f \cup \text{CFS}_p = \text{CFS} \).

[OP1] \( (F)(G)(F, G \in \text{CFS} \Rightarrow (F \preceq G \lor G \preceq F)) \).

Character and Letter Sequences

With the definitions developed above it is possible to define the perceptual objects with which the present research is most directly concerned: character sequences. A character sequence is simply an ordered set or sequence of cfs. A letter in the RS font, for example, is a one-place cfs sequence \( \leftrightarrow \) where \( \in \) RS. A sequence of letters in the RS font is similarly defined as an n-place cfs sequence where every member of the sequence is also a member of the set RS. But letters are not the only or the most general kind of cfs sequence. In fact, the set of primitives employed in the present research can be used to construct a set of \( 2^{14} \) (16,384) fully specified one-place cfs, of which only 26 are elements of RS and a set of \( 3^{14} \) (nearly 48 million) distinguishable one-place partially specified cfs!
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[D18] \( f \) is an \( n \)-place character sequence \( \leftrightarrow \)
\( f \) is an \( n \)-place sequence \& (\( y \in \text{Range}(f) \rightarrow y \in \text{CFS} \)).

Two conditions make up the definitions of this definition. The first condition is that a character sequence must be a sequence. The second condition is that every object that occupies a place in a character sequence (the range of the sequence) must be a cfs.

Two additional sets of interest result from this definition. CS will hereafter be taken to represent the set of all character sequences and \( \text{CS}^n \) is taken to represent the set of all \( n \)-place character sequences. Superscripts of "m", "s", and "c" will, as before, be used to specify material, sensory, and cognitive character sequences, and subscripts of "f" and "p" will indicate full and partial specification.

As above, principles of distinctness and completeness can be defined. A new ordering principle (OP2) based on the relative degree of featural specification of two character sequences is also introduced. What \( F \preceq G \) means is that the total number of features in the character sequence \( F \) is less than the total number of features in the character sequence \( G \); in other words, the feature description of \( F \) is less complete than that of \( G \).

[DP3] \( \text{CS}_f \cap \text{CS}_p = \emptyset \).

[CP2] \( \text{CS}_f \cup \text{CS}_p = \text{CS} \).

[OP2] \((n)(F)(G)(F,G \in \text{CS}^n \rightarrow ((F \preceq_c G) \leftrightarrow (\Sigma_{i=1}^n \text{Card}(F_i) \leq \Sigma_{i=1}^n \text{Card}(G_i))))\).

With character sequences defined, it is now possible to specify a variety of special kinds of character sequences that are of interest. A one-place character sequence is referred to as a character. Character sequences composed of feature sets from the Rumelhart/Siple font are referred to as letter sequences ([D19]). One-place letter sequences are referred to as letters.

[D19] \( f \) is a letter sequence \( \leftrightarrow \)
\( f \in \text{CS} \& (k)(k \in \text{Domain}(f) \rightarrow (\exists A)(A \in \text{RS} \& f_k \subseteq A)) \).

[D19] specifies two conditions that must be satisfied if an object is to be a letter sequence. One condition is that the object must be a character sequence. The second condition is that the set of feature primitives in every set that is a \( k \)-th member of \( f \) must be a subset of some set in \( \text{RS} \). What this means is that while a letter sequence may be less than fully specified, something is a letter sequence only if the sets of features it includes are at least "compatible with" (i.e., subsets of) the sets specified in \( \text{RS} \). Any character sequence therefore, that includes one or more sets of feature primitives that are not subsets of a set in \( \text{RS} \) is not a letter sequence.

Relations Between Objects Across Domains

The purpose of the preceding sections has been to specify a variety of basic objects with which the present theory is concerned. In addition, a variety of relations have been identified as holding between those objects within their respective domains. In this section relations between objects across domains are considered. Foremost among relations across domains are those that are referred to as unifiability relations.
Unifiability Relations

A unifiability relation is a relation that specifies how an object in one domain can be matched with an object in another domain. Since perception can be broadly defined as a matching of material objects with cognitive objects, it is apparent that unifiability relations are central to the theory proposed here. The first and simplest unifiability relation that is defined is the primitive unifiability relation.

A relation is a set of ordered pairs. Each of the ordered pairs that is a member of the primitive unifiability relation is referred to as a primitive unifier. A primitive unifier satisfies the following definition:

[D20]  \( x \) is a primitive unifier if

\[(\exists y)(\exists z)(x = \langle y, z \rangle \& y \text{ and } z \text{ are feature primitives } \& (|y| = |z|)),\]

where the term \(|\alpha|\) stands for the un-superscripted notational form of the originally domain-specific term \(\alpha\). (For example, \(|f^0| = f^0\), \(|a^3| = a^3\), and \(|\{f^0, a^2, f^3\}| = \{f^0, a^2, f^3\}\).) The set of all primitive unifiers constitutes the primitive unifiability relation \(UR_P\).

But the present theory is not only concerned with relations among primitive terms. A number of higher-level constructs such as the cfs and the character sequence have been developed. Unifiers and unifiability relations will also be defined for these higher-level objects. One higher-level unifiability relation is the cfs unifiability relation \((UR_{cfs})\) that specifies how cfs can be matched across domains. The elements of \(UR_{cfs}\) are referred to as cfs unifiers. Two kinds of cfs unifiers are defined. [D21] defines the most general kind of cfs unifier that requires that the second of the cfs paired be a subset of the first. [D22], on the other hand, defines a literal cfs unifier that satisfies the stricter criterion of set equivalence rather than set inclusion.

[D21]  \( x \) is a cfs unifier if

\[(\exists y)(\exists z)(x = \langle y, z \rangle \& y, z \in \text{CFS} \& \ |z| \subseteq |y|).\]

[D22]  \( x \) is a literal cfs unifier if

\[x \text{ is a cfs unifier } \& \ (\exists y)(\exists z)(x = \langle y, z \rangle \& \ |z| = |y|).\]

A second higher-level unifiability relation is the unifiability relation \((UR_{cs})\) that relates character sequences (i.e., elements of CS) to one another. The elements of \(UR_{cs}\) are referred to as cs unifiers and literal cs unifiers.

[D23]  \( x \) is a cs unifier if

\[(\exists A)(\exists B)(x = \langle A, B \rangle \& \ (\exists n)(A, B \in \text{CS}^n \& \ (i)(i \leq n \rightarrow < |A_i|, |B_i| > \text{ is a cfs unifier}))).\]

[D24]  \( x \) is a literal cs unifier if

\[(\exists A)(\exists B)(x = \langle A, B \rangle \& \ (\exists n)(A, B \in \text{CS}^n \& \ (i)(i \leq n \rightarrow < |A_i|, |B_i| > \text{ is a literal cfs unifier}))).\]
For the purposes of the present research, UR_{ln} is the highest-level unifiability relation required. Moreover, since the empirical objects of study in the present circumstances have been defined as sequences (neither features nor cs are sequences) the formal apparatus developed below relies almost exclusively on the UR_{ln} relation.

**Sensory Relations**

The purpose of the present section is to define sensory relations. In broadest possible terms, a sensory relation is a set of ordered pairs where the first of the two objects paired is a material object and the second is a sensory object. Several kinds of sensory relations are defined. The ultimate purpose of defining these various sensory relations is to provide a basis for defining a concept that is central to the present research, that of a sensory system. In formal terms,

[D25] \( f \) is a sensory relation \( \leftrightarrow \)
\[
(\exists x)(x \in f \to (\exists y)(\exists z)(x = <y,z> \land y \in U_m \land z \in U^s)).
\]

**The Primitive Sensory Transform Relation**

One sensory relation of interest is the primitive sensory transform relation that pairs material primitives with sensory primitives that are admissible representations of them. Sensory representations of material primitives are referred to as sensory transforms of those material primitives.

[D26] specifies which material-sensory pairs are members of this relation and thus defines how sensory representations are linked to material primitives. [D27] defines sensory transforms in terms of the sensory transform relation.

[D26] \( f \) is a primitive sensory transform relation \( \leftrightarrow \)
\[
f \text { is a sensory relation} \land (\forall x)(x \in f \to x \text { is a primitive unifier}).
\]

[D27] \( y \) is a primitive sensory transform of \( x \) \( \leftrightarrow \)
\[
(\exists f)(f \text { is a primitive sensory transform relation} \land <x,y> \in f).
\]

**The Stimulus Transform Relation**

According to the present theory, the material objects actually presented in experimental trials (i.e., stimuli) are not isolated feature primitives. A stimulus is always a character sequence, even if that sequence has only one character in it. It is, therefore, necessary to define transforms for the character sequences that are actually presented as stimuli in experimental trials. [D28] and [D29] provide the required definitions, where "is presented in an experimental trial" is a primitive concept that is assumed without definition.

[D28] \( x \) is a stimulus \( \leftrightarrow x \in CS_m \land x \) is presented in an experimental trial.

[D29] \( y \) is a stimulus transform of \( x \) \( \leftrightarrow x \) is a stimulus \& <x,y> \in UR_{ln}.\]
Complete and Incomplete Stimulus Transforms

Since sensory transforms of material stimuli need only satisfy a subset relation, it is apparent that a sensory transform can result in a loss of information. In other words, stimulus transforms are subject to a kind of degradation. For this reason, it is important to have a means of distinguishing complete stimulus transform pairs (i.e., no loss of information) from incomplete stimulus transform pairs (information lost). These distinctions are provided by definitions [D30] and [D31].

[D30] B is a complete stimulus transform of A \iff B is a stimulus transform of A & \langle A, B \rangle is a literal cs unifier.

[D31] B is an incomplete stimulus transform of A \iff B is a stimulus transform of A & \langle A, B \rangle is not a literal cs unifier.

It is also useful to define the degree of a stimulus transform, which is interpreted as the extent to which the information present in the stimulus is unambiguously represented in the transform.

[D32] B is an r-degree stimulus transform of A \iff B is a stimulus transform of A & (i \in \text{Domain}(A) \rightarrow

\exists P_i = \text{Card}(B_i) & P = \Sigma P_i & (\exists Q_i = \text{Card}(A_i) & Q = \Sigma Q_i) &

r = P/Q.

Sensory Output Functions

One of the consequences of the definitions provided above is that any one material stimulus can result in many possible sensory transforms. Since a material input can result in any one of a number of stimulus transforms, what mechanism will account for the output actually generated? In order to account for the actual performance of a sensory system it is necessary to define sensory output functions.

The Primitive Sensory Output Function

Formally, the primitive sensory output function is a function that assigns probability values to each of the feature primitives in a stimulus. The probability values assigned are interpreted as representing the likelihood that the material primitive will be represented in any given sensory transform. Conversely, one minus the probability assigned to a feature primitive is the probability that the information provided by the primitive is lost during sensory transformation.

[D33] f is a primitive sensory output function \iff

f: \{ <S, i, x>: S is a stimulus & i \in \text{Domain}(S) & x \in S_i \} \rightarrow [0,1].
The Stimulus Output and Transform Probability Functions

Since a stimulus is a complex of material feature primitives, the probability that a given stimulus will result in a given sensory output is related to the probabilities assigned to each of the feature primitives involved. There are, however, at least two ways the probability of a stimulus output can be related to the probabilities of feature primitives.

According to the assumption of featural independence, feature primitive probabilities are assigned independently of one another and the probability that any given sensory transform is output by the system is given by the product of the individual feature primitive probabilities that make up the stimulus. Alternatively, the assumption of featural dependence claims that feature primitive probabilities are dependent on one another in some fashion. According to the featural dependence assumption stimulus output probabilities depend on conditional probabilities.

An assumption of featural independence is simpler than that of featural dependence and provides both a plausible foundation for understanding primitive sensory processes (via the primitive sensory output function) and a straightforward mechanism for linking those primitive sensory events to higher-order sensory events. An assumption of featural independence is, therefore, adopted (as it was by McClelland and Rumelhart (1981), who do not include any feature-feature connections) in [D34] where the concept of a stimulus transform probability function ($P_{ST}$) is defined.

[D34] \[ P_{ST} \text{ is a stimulus transform probability function} \leftrightarrow \]

\[ (\exists x)(x \in CS^n) \land (\exists y)(y \in CS^o) \land \]

\[ (\exists f)(f \text{ is a primitive sensory output function}) \land \]

\[ P_{ST}(x,y) = \prod_{x \in \text{Domain}(x)} \left[ \prod_{z \in x} \left\{ \begin{array}{ll}
   f(x_i,z) & \text{if } z \in y_i \\
   1 - f(x_i,z) & \text{if } \neg(z \in y_i)
\end{array} \right. \right] \]

[D35] \[ F_{st} \text{ is a stimulus output function} \leftrightarrow \]

\[ F_{st}(\{S: S \text{ is a stimulus}\}) \rightarrow \{T: T \text{ is a stimulus transform of } S\}. \]

Sensory Systems

A sensory system is simply a stimulus output function that is coupled with a stimulus transform probability function $P_{ST}$. Specific individual sensory systems are, according to the present theory, defined completely by a specific primitive sensory output function since $P_{ST}$ is a derivative of the primitive sensory output function.

[D36] \[ S \text{ is a sensory system} \leftrightarrow \]

\[ (\exists P_{st})(\exists F_{st})(P_{ST} \text{ is a stimulus transform probability function} \land \]

\[ F_{st} \text{ is a stimulus output function} \land S = \langle P_{st}, F_{st} \rangle). \]
Cognitive Relations

All of the relations that have been defined to this point have been sensory relations; they have been limited to those between material and sensory objects. The purpose of the present section is to define some additional cognitive entities and to begin to define cognitive relations that exist between sensory and cognitive objects. Just as one of the primary goals of the sensory relations section was to provide a formal account for the concept of a sensory system, a major objective in the present section is to provide a formal account for the concept of a cognitive system.

As is true of a sensory system, a cognitive system is founded upon special kinds of entities and relations. The entities upon which a cognitive system is founded are objects constructed from cognitive primitives. The relations upon which a cognitive system are founded are referred to as cognitive relations. The purpose of cognitive relations is to explain how cognitive objects are related to sensory and (indirectly) material objects. In particular, the present theory seeks to explain two kinds of psychological events, learning and recognition. Recognition, as indicated above, is conceived of as a matching of a material stimulus with a cognitive entity that is an element of existing cognitive set. Learning is conceived of as the addition of a cognitive entity to an existing cognitive set or the modification of existing cognitive entities. In order to define recognition and learning, the cognitive entities and sets referred to above must be defined and that is the first objective of this section.

Identification Principles and Perceptual Repertoires

The sets of cognitive objects identified above are referred to as perceptual repertoires. The elements of a perceptual repertoire are referred to as identification principles. An identification principle corresponds to the concept of perceptual "knowledge"; i.e., the kind of knowledge we acquire when we learn to recognize a visual pattern.

An identification principle is an ordered pair. One of the two items in the pair represents a response. The second of the two items in the ordered pairs represents the feature pattern associated with the response. A perceptual repertoire is a set of ordered pairs, each of which is an identification principle.

[D37] x is an identification principle (IP) ↔

(∃y)(∃z)(y is a response & z ∈ CS & x = <y,z>).

[D38] f is a perceptual repertoire (PR) ↔ (x)(x ∈ f → x is an IP).

Like other higher-order cognitive entities, identification principles and perceptual repertoires can be characterized as either fully specified or partially specified. Subscripts of "f" and "p" serve to identify fully and partially specified sets of higher-order cognitive entities.

Identification principles and perceptual repertoires can also be categorized as either RS-consistent or RS-inconsistent. RS-consistency serves, in the present context, as a dichotomous measure of the "correctness" or "incorrectness" of the association(s) represented in an IP (or PR) with respect to the standard established by the Rumelhart and Siple font (i.e., the set RS). When an IP is RS-consistent it means that the association
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encoded in that IP adheres to the RS standard; in other words, the knowledge being applied by the system is correct. Conversely, an IP is said to be RS-inconsistent when it does not adhere to the RS standard; the learning that has taken place is inappropriate. A PR is said to be RS-inconsistent when at least one of the IPs in it is RS-inconsistent. As will become apparent in Part 2 of this paper, the concepts of completeness and consistency defined here have important epistemological and pedagogical implications for letter and word perception and acquisition.

The Cognitive Match Relation and Cognitive Match Probability Function

A cognitive relation is a set of ordered pairs where the first of the two objects paired is a sensory object and the second is a cognitive object.

[D39] \( f \) is a cognitive relation \( \iff \)

\[(x)(x \in f \implies (\exists y)(\exists z)(x = <y, z> \land y \in U^y \land z \in U^z)).\]

It is the purpose of the cognitive relations defined below to lead to the concept of a cognitive system. With a cognitive system defined, it will then be possible to define a perceptual system, which represents the highest-level object with which the present research is concerned. Two kinds of cognitive relations are of special interest, the cognitive match relation [D40] and the cognitive match probability function [D42].

The cognitive match relation is a set of ordered pairs each of which is composed of a sensory transform of a material stimulus and an element of a perceptual repertoire that has as one of its elements a cognitive object unifiable with the sensory transform.

[D40] \( f \) is a cognitive match relation \( \iff \)

\[f \subseteq \{<s, <y, z>> : (\exists A)(A \text{ is a stimulus} \land s \text{ is a sensory transform of } A \land <y, z> \text{ is an identification principle} \land (\exists k)(s \epsilon CS^i \land z \epsilon CS^k) \land (i \leq k \implies (i \epsilon \text{ Domain}(s) \implies (|s| \cup |z| \epsilon CFS)) \lor (k \leq i \implies (i \epsilon \text{ Domain}(z) \implies (|s| \cup |z| \epsilon CFS)))\}.

[D41] \( x \) is a cognitive match for \( y \) \( \iff \)

\[(\exists M)(M \text{ is a cognitive match relation} \land (\exists PR)(PR \text{ is a perceptual repertoire} \land (\exists z)(\exists w)(z = <y, <w, x>>) \land z \epsilon M \land <w, x> \epsilon PR)).\]

The set of all cognitive matches for an entity \( A \) is represented by \( CM_A \) and the set of all cognitive matches for an entity \( A \) in some perceptual repertoire \( R \) is represented by \( CM_{A|R} \).
One aspect of cognitive matching that has not yet been addressed, however, is how a cognitive match is actually selected when two or more possible matches are available in a perceptual repertoire. When two or more possible matches exist a match is selected according to a cognitive match probability function ([D42]) that assigns probability values to each of the possible matches in the perceptual repertoire.

\[ f: \{ A, PR, B \mid A \text{ is a stimulus} \land PR \text{ is a perceptual repertoire} \land B \in CM_{A \rightarrow PR} \} \rightarrow [0,1]. \]

One characteristic of [D40] worthy of special attention is that, unlike the sensory transform relation that is based on the subset relation, the cognitive match relation is based on the concept of logical unification. One consequence of unification-based matching is that a fully specified cognitive match can be paired with a partially specified sensory transform, and vice versa. The result is that cognitive matching can introduce information to the perceptual process and can even result in a form of incidental learning. Recall that sensory transforms of a stimulus must satisfy a subset requirement that means sensory transforms can never be more completely specified than the original material input. The cognitive match relation, however, is not bound by this kind of requirement. Fully specified sensory transforms can be matched with partially specified identification principles, and partially specified sensory transforms can be matched with fully specified identification principles. Whereas sensory transforms of material stimuli are subject to a completeness upper-bound determined by the original material input, no such constraint is assumed to operate at the cognitive level. In fact, cognitive matching can actually introduce new information to the perceptual process. The implication is that perception is not simply a passive processing of information from outside, but rather an interaction of information from outside with information already represented within the system.

In addition, the assumption that a cognitive system can match less fully specified cognitive units with more fully specified sensory inputs means the matching process can elaborate on the content of cognitive units. This elaboration can result in modifications to existing identification principles, and will hereafter be referred to as the principle of cognitive elaboration. This principle can be shown to be a powerful explanatory mechanism for a variety of observed perceptual phenomena. Moreover, it is not only consistent with, but also helps explain how the rather severe constraints imposed by the subset requirement for sensory transforms can accommodate the well-known abilities of human subjects to recognize highly degraded but familiar stimuli.

Whereas the cognitive match relation and its related probability function serve to explain the principles of recognition, learning in the system has not yet been defined, that is the purpose of [D43].

The Cognitive Development Function

[D43] defines a specialized learning function that relates a sensory transform/perceptual repertoire pair \(<s, PR_1>\) with a second perceptual repertoire \((PR_2)\) that corresponds to the final state of the repertoire after matching of the sensory
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transform s. The function identified in [D43] introduces the theory of learning proposed in the present research and will hereafter be referred to as a cognitive development function.

[D43] f: a cognitive development function \iff f is a function &

\[ \text{Domain}(f) = \{<S, PR_1>: S \text{ is a sensory transform } \&\]

PR_1 is a perceptual repertoire} &

\[ \text{Range}(f) = \{<PR_2>: PR_2 \text{ is a perceptual repertoire} } \&\]

\( (S)(PR_1)(PR_2)(<S, PR_1> \in \text{Domain}(f) \implies \]

1 \( (CM_{\text{SinPR}_1} = \emptyset \& (\exists C)(<S, C> \text{ is a literal cs unifier } \&\]

\( (\exists r)(r \text{ is a response } \& PR_2 = PR_1 \cup \{<r, C>\}) \vee \]

2 \( (\exists !<p, q>)(<p, q> \in CM_{\text{SinPR}_1} \& <p, q> \text{ is the cognitive match }\]

selected according to the cognitive match probability function & PR_2 = \{(PR_1 - \{<p, q>\}) \cup \{<p, |S| \cup |q|>\}).

The consequent of [D43] is rather complex but boils down to a specification of two cases (identified by 1 and 2), the first of which accounts for additions to the perceptual repertoire and the second of which accounts for modifications to existing identification principles in the perceptual repertoire. The first case applies when there is no cognitive match for a sensory input in a perceptual repertoire. In this case the literal cognitive equivalent of the sensory transform is introduced as a new member of the perceptual repertoire PR_2. Case 1 corresponds to the traditional conception of learning as the association of a response with a stimulus.

The second case occurs when there is a cognitive match in PR_1 for the sensory input. In this case, the cognitive match ultimately selected may be modified as a consequence of being selected. If the sensory input provides less than or the same information already present in the cognitive match in PR_1, the IP selected is preserved unchanged. If, on the other hand, the sensory information exceeds that represented in the cognitive representation, then the additional information present in the sensory transform is added to that in the original match and the new IP that results takes the place of the original matching unit in PR_2. Case 2 introduces the concept of unification-based learning, where learning occurs not by adding an association to a perceptual repertoire but through the modification of existing identification principles.

Cognitive Systems

A cognitive system can now be defined in terms of the concepts of perceptual repertoire, cognitive match probability function, and the cognitive development function.
C is a cognitive system $\leftrightarrow (\exists PR)(\exists F_{cm})(\exists F_{cd})(C = <PR; F_{cm}; F_{cd}>)$ &

PR is a perceptual repertoire & $F_{cm}$ is a cognitive match probability

function & $F_{cd}$ is a cognitive development function).

Moreover, with the concepts of cognitive system and sensory system defined it is now possible to define the concept of a perceptual system.

**Perceptual Systems**

Informally, a perceptual system is the object that results when a sensory system that transforms material stimuli into sensory transforms is combined with a cognitive system that matches sensory transforms with existing cognitive entities, and in the process, may modify itself. In formal terms,

[D45] $f$ is a perceptual system $\leftrightarrow$

$((\exists P_{st})(\exists F_{st})(\exists PR)(\exists F_{cm})(\exists F_{cd})(f = <P_{st}; F_{st}; PR; F_{cm}; F_{cd}>)$ &

$<P_{st}; F_{st}>$ is a sensory system & $<PR; F_{cm}; F_{cd}>$ is a cognitive system).

**Summary**

This paper defines the formal foundation for a theory of letter and word perception and learning. According to the theory, perception of printed material is mediated by a sensory system that creates sensory transforms that are matched with cognitive representations residing in long-term memory. Matching of sensory transforms with cognitive representations is assumed to occur by means of logical unification. Logical unification appears to provide a powerful explanatory mechanism capable of accounting for top-down perceptual effects. Logical unification also appears to provide a foundation for a developmental theory of perceptual learning based on cognitive elaboration. The proposed foundation appears to offer a broader and more comprehensive base for both developmental and pedagogical aspects of letter and word perception and acquisition. It is the purpose of Part 2 of this paper to show how this foundation is implemented as a Prolog computer simulation and, further, how it leads to a number of developmental and pedagogical consequences that could have implications for future theory and instructional practice.

**References**


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