1. Find and sketch the asymptotes of the function
\[ y = \frac{4 + 2x}{1 + x}. \]

A: The vertical asymptotes are where the denominator vanishes, thus
\[ x = -1, \quad (4 \text{pts}) \]
and then
\[ \lim_{x \to -1^-} \frac{4 + 2x}{1 + x} = -\infty, \quad \lim_{x \to -1^+} \frac{4 + x}{1 + x} = \infty. \quad (6 \text{ pts}) \]

The horizontal asymptote is the \( y \)-value when \( x \) tends to infinity or negative infinity, thus
\[ y = 2, \quad (4 \text{ pts}) \]
since
\[ \lim_{x \to \pm\infty} \frac{4 + 2x}{1 + x} = \lim_{x \to -\infty} \frac{4 + 2}{x + 1} = 2. \quad (6 \text{ pts}) \]
2. Find the indefinite integral

\[ \int \frac{a}{e^{-x/b}} \, dx. \]

A: First, we rewrite the integral as \( a \int e^{x/b} \, dx \). Then we let \( u = x/b \) and so

\[ u' = 1/b, \quad du = (1/b)\,dx, \quad \text{and} \quad dx = b\,du, \quad \text{(8 pts)} \]

and the integral is

\[ a \int e^{x/b} \, dx = ab \int e^u \, du = ab e^u + C \quad \text{(8 pts)} \]

\[ = abe^{x/b} + C. \quad \text{(4 pts)} \]

You may also choose \( u = -x/b \) or even \( u = e^{-x/b} \), but using either one is a bit longer.

3. Find the definite integral

\[ \int_1^5 \frac{\ln x}{x} \, dx. \]

A: First, we need to find the antiderivative \( F(x) \) of \( \ln x/x \).

To this end let \( u = \ln x \) then

\[ u' = \frac{1}{x}, \quad du = \frac{1}{x} \, dx, \quad \text{(6 pts)} \]

and then the integral of \( \ln x/x \) can be written

\[ F(x) = \int \frac{\ln x}{x} \, dx = \int u \, du = \frac{1}{2} u^2 + C = \frac{1}{2} (\ln |x|)^2 + C. \quad \text{(8 pts)} \]

Using now the Fundamental Theorem of Calculus we have

\[ \int_1^5 \frac{\ln x}{x} \, dx = F(5) - F(1) = \frac{1}{2} (\ln 5)^2 = 1.3. \quad \text{(6 pts)} \]

Here, we used the fact that \( \ln 1 = 0 \).
4. Find the definite integral
\[ \int_0^a \frac{1}{x+b} \, dx. \]

A: First, we need to find the antiderivative \( F(x) \) of \( 1/(x+b) \).
To this end let \( u = x + b \) then
\[ u' = 1, \quad du = dx, \quad (6 \text{ pts}) \]
and then the integral of \( 1/(x+b) \) can be written as
\[ F(x) = \int \frac{1}{x+b} \, dx = \int \frac{1}{u} \, du = \ln |u| + C = \ln |x+b| + C. \quad (8 \text{ pts}) \]

Using now the Fundamental Theorem of Calculus we have
\[ \int_0^a \frac{1}{x+b} \, dx = F(a) - F(0) = \ln |b+a| - \ln |b| = \ln \left| \frac{b+a}{b} \right|. \quad (6 \text{ pts}) \]

5. Find the indefinite integral
\[ \int \frac{2xe^{x^2}}{e^{x^2} - b} \, dx. \]

A: We choose
\[ u = e^{x^2} - b, \quad (4 \text{ pts}) \]
and then
\[ u' = 2xe^{x^2}, \quad du = 2xe^{x^2} \, dx. \quad (8 \text{ pts}) \]

Therefore,
\[ F(x) = \int \frac{2xe^{x^2}}{e^{x^2} - b} \, dx = \int \frac{du}{u} = \ln |u| + C = \ln |e^{x^2} - b| + C. \quad (8 \text{ pts}) \]
6. The cost per hour (in dollars) of operating a certain car is given by

\[ C = 0.6s - 0.006s^2 + 0.1, \quad \text{for} \ 0 \leq s \leq 65, \]

where \( s \) is the speed in mph. At what speed is the cost \( C \) minimum? Maximum?

A: We need to find the critical points of the function \( C \). We have

\[ C' = 0.6 - 0.012s, \]

and \( C' = 0 \) when \( s = 50 \) and, since \( C'' = -0.012 < 0 \), the function is concave down (12 pts). Next,

\[ C(50) = 15.1, \quad C(0) = 0.1, \quad C(65) = 13.75 \quad (6 \text{ pts}) \]

We conclude that the cost in minimum when \( s = 0 \), and is maximum when \( s = 50 \). (2 pts)

7. (You have to answer this question.) A manufacturer has to design a container in the form of a rectangular box with a square base, no top, and with volume \( V = 256 \). What are the dimensions of the box that require the least amount of material?

A: Since the base of the box is a square, if we denote its length by \( x \) and the height of the box by \( y \) we have that the volume \( V \) of the box is

\[ V = x^2y = 256. \]

Therefore, \( y = 256/x^2 \) (2 pts). Next, the area of the box consists of the basis, \( x^2 \) and four sides with area \( xy \) each. Thus, the area \( S \) of the box is

\[ S = x^2 + 4xy = x^2 + 4x \frac{256}{x^2} = x^2 + \frac{1024}{x}. \quad (3 \text{ pts}) \]

To find the minimum we find the critical points, so

\[ S' = 2x - \frac{1024}{x^2}, \quad S'' = 2 + \frac{2048}{x^3} > 0. \quad (3 \text{ pts}) \]

Then \( S' = 0 \) at \( x = 8 \), and it is a minimum point since the function is concave up. We conclude that the dimensions of the desired box are \( x = 8 \) and \( y = 4. \) (2 pts)
A manufacturer’s marginal-cost function is
\[
\frac{dc}{dq} = 0.6q^2 - 0.2q + 10.
\]
Determine the cost involved to increase production from 50 to 60 units.

A: We are given the marginal-cost function \(c'(q)\), which is the derivative of the cost function \(c(q)\), and asked about the cost involved in increasing \(q\) from 50 to 60, that is we need to compute \(c(60) - c(50)\). Thus, we need to calculate

\[
c(60) - c(50) = \int_{50}^{60} \frac{dc}{dq} dq = \int_{50}^{60} (0.6q^2 - 0.2q + 10) \, dq. \quad (2 \text{ pts})
\]

First, we find the antiderivative, \(c\),

\[
c(q) = \int (0.6q^2 - 0.2q + 10) \, dq
\]

\[
= 0.6 \int q^2 \, dq - 0.2 \int q \, dq + 10 \int \, dq. \quad (2 \text{ pts})
\]

Now, we have (here \(C\) is the constant of integration),

\[
0.6 \int q^2 \, dq = 0.6 \cdot \frac{1}{3} q^3 + C = 0.2q^3 + C, \quad (1 \text{ pt})
\]

\[
0.2 \int q \, dq = 0.2 \cdot \frac{1}{2} q^2 + C = 0.1q^2 + C, \quad (1 \text{ pt})
\]

\[
10 \int \, dq = 10q + C. \quad (1 \text{ pt})
\]

Thus,

\[
c(q) = 0.2q^3 - 0.1q^2 + 10q + C. \quad (3 \text{ pts})
\]

Finally, the answer is

\[
c(60) - c(50) = \left(0.2(60)^3 - 0.1(60)^2 + 10(60)\right)
\]

\[
- \left(0.2(50)^3 - 0.1(50)^2 + 10(50)\right)
\]

\[
= 43440 - 25,250 = 18,190. \quad (2 \text{ pts})
\]